

文章编号:1000-8055(2011)04-0727-08

Estimation method for random sonic fatigue life of thin-walled structure of a combustor liner based on stress probability distribution

SHA Yun-dong¹, GUO Xiao-peng², LIAO Lian-fang², XIE Li-juan²

(1. Liaoning Key Laboratory of Digital Technology Simulation and Test Technique,
College of Power and Energy Engineering,
Shenyang Aerospace University, Shenyang 110136, China;
2. Shenyang Aeroengine Research Institute,
Aviation Industry Corporation of China, Shenyang 110015, China)

Abstract: As to the sonic fatigue problem of an aero-engine combustor liner structure under the random acoustic loadings, an effective method for predicting the fatigue life of a structure under random loadings was studied. Firstly, the probability distribution of Von Mises stress of thin-walled structure under random loadings was studied, analysis suggested that probability density function of Von Mises stress process accord approximately with two-parameter Weibull distribution. The formula for calculating Weibull parameters were given. Based on the Miner linear theory, the method for predicting the random sonic fatigue life based on the stress probability density was developed, and the model for fatigue life prediction was constructed. As an example, an aero-engine combustor liner structure was considered. The power spectrum density (PSD) of the vibrational stress response was calculated by using the coupled FEM/BEM (finite element method/boundary element method) model, the fatigue life was estimated by using the constructed model. And considering the influence of the wide frequency band, the calculated results were modified. Comparative analysis shows that the estimated results of sonic fatigue of the combustor liner structure by using Weibull distribution of Von Mises stress are more conservative than using Dirlik distribution to some extent. The results show that the methods presented in this paper are practical for the random fatigue life analysis of the aeronautical thin-walled structures.

Key words: thin-walled structure; sonic fatigue; life estimation; probability density function (PDF); power spectrum density (PSD)

CLC number: V232.6; V215.5

Document code: A

Introduction

Nowadays advanced aircraft and spacecraft structures are exposed to increasingly severe operating environments, including a combination of mechanical, pressure, acoustic and thermal loads. These loading conditions can cause struc-

tures to exhibit complex response characteristics. For thin-walled structures subjected solely to high intensity acoustic loadings, random vibration response can be caused, and as a result, sonic fatigue, as a high cycle fatigue, can cause cracks in structure. To help ensure structural integrity, an evaluation of random sonic fatigue

Received: 2010-11-15; **revision received:** 2011-03-07

Foundation item: Supported by the National Aviation Fundamental Science Foundation of China (No. 02C54007)

E-mail: SHA Yun-dong (professor): ydsha2003@vip.sina.com

GUO Xiao-peng (research fellow): guoxiaopeng312@163.com

life is required. Generally, the total fatigue damage may be determined by using the Palmgren-Miner rule. S-N curves are usually determined for uni-axial stress states. However, in a real environment, stresses will not be uni-axial, but bi-axial or in the most general case multi-axial. Many of them are based on the critical plane approach. Alternatively, it is possible to calculate an equivalent stress from the multi-axial stresses, which can subsequently be used to perform uni-axial stress-life fatigue analysis as described above. Popular used equivalent stresses are Von Mises stress, the maximum principal stress and the maximum shear stress. The Von Mises stress is a non-Gaussian random process whose probability distribution is hard to determine, even in linear structures under Gaussian excitations. This makes it difficult to estimate the probability distribution of the extremes of Von Mises stress. Techniques for obtaining approximations for outcrossing probabilities of non-Gaussian processes have been discussed in the Refs. [1-2]. Grigoriu obtained approximate estimates for the mean outcrossing rate of non-Gaussian translation processes by studying the outcrossing characteristics of a Gaussian process obtained from Nataf's transformation of the parent non-Gaussian process^[3]. Segalman et al have derived and presented an expression for the cumulative probability distribution for Von Mises stress resulting from random loadings that are Gaussian and of zero mean^[4-6]. This paper develops a methodology for estimating the random sonic fatigue life of thin-walled structure. The emphasis is put on the approximations for the probability distribution of extreme values for the Von Mises stress in linear structures under band-limited Gaussian white noise loads. For this purpose, the probability distribution of Von Mises stress of thin-walled structure under random loadings is investigated. Analysis suggests that probability density function of Von Mises stress process accords approximately with two-parameter Weibull distribution, the formula for calculating Weibull param-

eters and the broadband stochastic process Dirlik equation are given, as a result, the probability density function and the peak probability density function of Von Mises process can be determined. For a numerical example, applying the coupled FEM/BEM (finite element method/boundary element method) model, the vibrational stress response power spectral density (PSD) of a combustor liner under random acoustic loadings is calculated, and then, applying the method given in this paper, the fatigue life is predicted. The analysis results show that the method is suitable to the random fatigue life analysis of aero-engine structures.

1 Probability density of Von Mises stress

1.1 Probability density function for Von Mises stress

The fatigue crack usually appears on the surface of structure. If the surface is in the condition of plane stress, Von Mises stress can be defined as follows^[7]:

$$s_v^2 = s_x^2 + s_y^2 - s_x s_y + 3 s_{xy}^2 \quad (1)$$

Where s_v is Von Mises stress, s_x , s_y and s_{xy} are the normal and shear stress components respectively. When the random acoustic loading is Gaussian white noise, the stress components are zero mean Gaussian random processes.

How to accurately determine the peak value probability density function of stress response is a key factor in estimating the cumulative damage. Based on this, in Ref. [8], the probability distribution of Von Mises stress is verified to obey two-parameter Weibull distribution. When $s_v \geq 0$, The Weibull distribution function can be expressed mathematically as follows:

$$p(s_v) = \frac{m}{\eta} \left(\frac{s_v}{\eta} \right)^{m-1} \exp \left[- \left(\frac{s_v}{\eta} \right)^m \right] \quad (2)$$

Where $p(s_v)$ is the probability density function of Von Mises stress, m and η are Weibull Parameters.

The expected and variance can be expressed as:

$$E(s_v) = \eta \Gamma \left(1 + \frac{1}{m} \right) \quad (3)$$

$$\sigma^2(s_v) = \eta^2 \left[\Gamma \left(1 + \frac{2}{m} \right) - \Gamma^2 \left(1 + \frac{1}{m} \right) \right] \quad (4)$$

Where Γ is the Gamma function.

1.2 Peak value probability density function for Von Mises stress

Generally, the peak value probability density function (PDF) of a Gaussian random process $X(t)$ can be obtained by its joint probability den-

$$p(s) = \frac{\frac{1}{\eta} \left[\frac{s + E(s_v) - \sigma(s_v)}{\eta} \right]^{m-2} \exp \left\{ - \left[\frac{s + E(s_v) - \sigma(s_v)}{\eta} \right]^m \right\}}{\left[\Gamma \left(1 + \frac{1}{m} \right) \right]^{m-1} \exp \left\{ - \left[\Gamma \left(1 + \frac{1}{m} \right) \right]^m \right\}} \cdot \left\{ m \left[\frac{s + E(s_v) - \sigma(s_v)}{\eta} \right]^m - m + 1 \right\} \quad (5)$$

2 Dirlik equation for broad band processes

2.1 Spectral moments and bandwidth

PSD function $G(f)$ is a description of stationary random process in frequency domain. The n -order spectral moments of PSD function m_n are defined as follows:

$$m_n = \int_{-\infty}^{+\infty} f^n G(f) df \quad (6)$$

For a Gaussian process, the expected rate of occurrence of peaks v_p (i. e. peaks per unit time) and the expected rate of mean up-crossings $V^+(S)$ (i. e. crossing with positive slope of the mean value per unit time) are also known:

$$v_p = \sqrt{\frac{m_4}{m_2}}, \quad V^+(S) = \sqrt{\frac{m_2}{m_0}} \quad (7)$$

In order to characterise the shape and the geometrical frequency of a given spectrum, it's practice to define bandwidth parameters as functions of spectral moments; the most used bandwidth parameters is β .

$$\beta = \frac{m_2}{\sqrt{m_0 m_4}} \quad (8)$$

2.2 Dirlik equation

Instead of using one Rayleigh distribution to represent the probability density function of peaks of a broad-band stress process, Dirlik analyzed dozens of different shapes of PSD function by numerical simulation, and proposed a semi-empirical model which is a mixture of one exponential and two Rayleigh distributions. This distribution was designed to approximate the stress

function. But for a non-Gaussian process, the joint probability density is difficult to get. It is discovered that the response of a weak damping system may be regarded as narrow band random process. Based on this, the peak value PDF of Von Mises stress can be expressed as^[9]:

range distribution for rain-flow counting $P_p(S)$. Dirlik derived his equation by fitting the shape of a rain-flow range distribution. The associated parameters were determined by minimising the normalized error between the rain-flow ranges and his model^[10-12].

Dirlik's formula is

$$P_p(S) = \frac{1}{2\sqrt{m_0}} \left[\frac{D_1}{Q} e^{-z/Q} + \frac{D_2}{R^2} e^{-z^2/(2R^2)} + D_3 z e^{-z^2/2} \right] \quad (9)$$

where

$$\begin{cases} D_1 = 2(x_m - r^2)/(1 + r^2) \\ D_2 = (1 - r - D_1 + D_1^2)/(1 - R) \\ D_3 = 1 - D_1 - D_2 \\ z = s/(2\sqrt{m_0}) \\ Q = 1.25(r - D_3 - D_2 R)/D_1 \\ R = (r - x_m - D_1^2)/(1 - r - D_1 + D_1^2) \\ r = m_2/\sqrt{m_0 \cdot m_4} \\ x_m = (m_1/m_0)\sqrt{m_2/m_4} \end{cases} \quad (10)$$

3 Weibull parameter estimation and Kolmogorov-Smirnov (K-S) test

3.1 Weibull parameter estimation

The cumulative distribution is obtained by integrating Eq. (2) and takes form:

$$F(s) = 1 - e^{-\left(\frac{s}{\eta}\right)^m} \quad (11)$$

The scale ' η ' and shape parameters ' m ' were estimated using the method of maximum likelihood estimation (MLE) as follows^[13]:

$$\hat{\eta} = \left[\left(\frac{1}{n} \right) \sum_{i=1}^n x_i^m \right]^{\frac{1}{m}} \quad (12)$$

$$\hat{m} = \left[\left(\sum_{i=1}^n x_i^{\hat{m}} \ln x_i \right) \left(\sum_{i=1}^n x_i^{\hat{m}} \right)^{-1} - n^{-1} \sum_{i=1}^n \ln x_i \right]^{-1} \quad (13)$$

3.2 Goodness of K-S test

In order to verify the goodness of the probability distribution model to stress data, the K-S test should be conducted. According to the K-S test method, the distribution function of the parent set X is defined as $F(x)$, while the empirical distribution function (EDF) is

$$F_n(x) = \begin{cases} 0 & x < x_1 \\ \frac{i}{n} & x_i \leq x < x_{i+1} \\ 1 & x \geq x_n \end{cases} \quad (14)$$

where i the cumulative frequency and n is the sample size. Define the statistical terms D_n as

$$D_n = \max_{1 \leq k \leq n} |F_n(x_k) - F(x)| \quad (15)$$

The critical value of $D_{n,\alpha}$ can be found in the critical value table of the K-S test. If $D_n \leq D_{n,\alpha}$, then the distribution fitting is good, otherwise the fitting is not satisfactory. Usually, the significant level α is taken to be 0.05.

4 Random sonic fatigue life estimation

4.1 Sonic fatigue life model

The fatigue life of components is determined by the material mechanical properties and stress level. The relationship between alternating stress amplitude S and number of cycles to failure N for many materials typically follows the S-N equation:

$$S^b N = C \quad (16)$$

Where b and C are material parameters.

From Palmgren-Miner linear cumulative damage theory, the fatigue life of the structure can be obtained:

$$N_t = TE[0] = \left[v_p \cdot \int_0^\infty \frac{P_p(S)}{N(S)} dS \right]^{-1} \cdot E[0] \quad (17)$$

Where N_t is the numbers of cycles to failure; v_p is speed of stress peaks; $E[0]$ is the number of upword mean crossings per second which is equal to $V^+(S)$; $P_p(S)$ represents the probability density function of stress cycles.

4.2 Wide-band revision of the fatigue life

As for the wide-band random vibration, attention should be paid to the influence of the partial peak values on the fatigue life. Wirching P H has corrected the fatigue life according to different shapes of the stress response PSD, and obtained formula for life estimation which is suitable for the wide-band random vibration.

$$N_{tl} = \frac{N_t}{\lambda} \quad (18)$$

Where N_{tl} represents total number of cycles to failure after the revision. The relation of the correction factor λ , material parameters m , irregular factor β is defined in the formula as follows^[14]:

$$\lambda = 0.926 - 0.033m + (0.074 + 0.033m)(1 - \sqrt{1 - \beta})^{(1.587m - 2.3323)} \quad (19)$$

5 Numerical example and discussion

Apply the method developed in this paper to estimate the acoustic fatigue life of an aero-engine combustor liner, which material is GH536, with mass density $\rho = 8.230 \times 10^3 \text{ kg/m}^3$, Poisson's ratio $\nu = 0.3$, Young's modulus $E = 1.855 \times 10^{11} \text{ Pa}$. Boundary conditions: according to installation form of the liner; the degree of freedom (DOF) of five positions on the heads of the liner are restrained in X, Y, Z direction, and the DOF except the axial direction on the rear end of the liner are restrained.

5.1 Acoustic load and S-N curve fitting

The acoustic load considered in this paper is the Gaussian band-limited white noise. The bandwidth is $8 \sim 1024 \text{ Hz}$, the sound pressure level (SPL) are 140 dB and 160 dB respectively. The material GH536 fatigue data are shown in Table 1. Using Basquin fatigue life model, the fatigue data are fitted to obtain the expression $S^{1.8648} N = 10^{7.9841}$. The correlation coefficient of fitting is 0.9993, so the fitting effect is better. It's reliable to estimate structure fatigue life using Basquin fatigue life model.

Table 1 TGH536 fatigue data

Variable	Data					
Stress amplitude/MPa	420	402	378	373	367	365
Fatigue life/ 10^5 cycles	0.5	1	5	10	50	100

5.2 Response of the combustor liner under band-limited white noise loadings

This paper calculates sonic fatigue stress of a combustor liner using the coupled FEM/BEM model. The FEM model of the liner is show in Fig. 1, which illustrates the typical nodes position selected for analyzing.

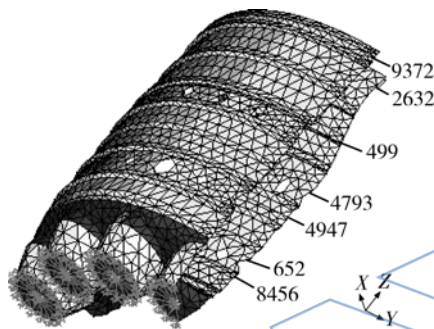


Fig. 1 FEM model and typical nodes position of the liner

Fig. 2 illustrates the Von Mises stress PSD at node 2632 (dilution air hole edge on inner skin) under sound pressure level 140 dB and 160 dB. Frequency spectrum analysis of the calculated stresses shows that there are many peak values in the liner stress response within the frequency band calculated. Comparing with the na-

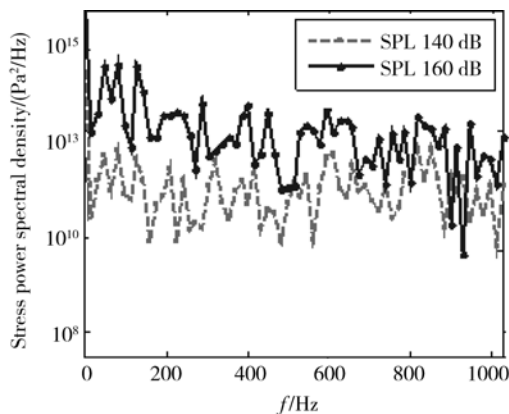


Fig. 2 Von Mises stress power spectral density at node 2632

ture frequencies of the structure, it can be found that the response to acoustic excitation is of multi-mode characteristics, therefore, the frequency construction of the response spectrum must be considered in anti-fatigue design.

Fig. 3 and Fig. 4 illustrate the mean square Von Mises stress at selected nodes under sound pressure level 140 dB and 160 dB. It can be seen that the influences of sound pressure level and nodes position to the mean square response. Moreover, for having used linear model, the stress responses under sound pressure level 140 dB and 160 dB have the similar frequency construction, but the amplitude value increasing in proportion to sound pressure.

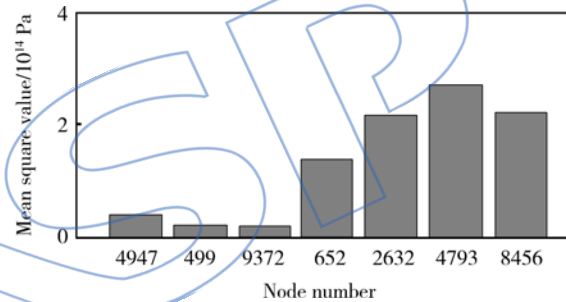


Fig. 3 Mean square Von Mises stress at selected nodes under sound pressure level 140 dB

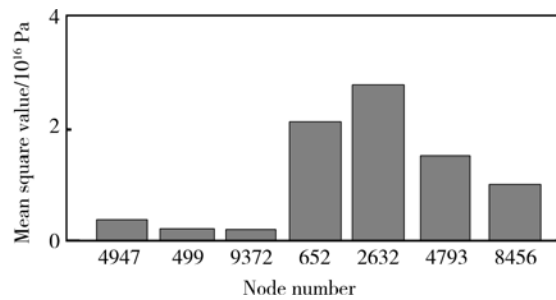


Fig. 4 Mean square Von Mises stress at selected nodes under sound pressure level 160 dB

5.3 Probability distribution of Von Mises stress response

These parameters values $\hat{m}$ and $\hat{\eta}$ are obtained by solving Eqs. (15) and (6) for estimating the shape and scale parameters by using maximum likelihood estimators. By calculation, the node 2632 under sound pressure level 160 dB which parameters values $\hat{m}$ and $\hat{\eta}$ are obtained. $m = 2.24214$, $\eta = 108.051$. Fig. 5 shows the

probability density function plots of Weibull distribution for the node with the histogram density plot superimposed on the PDF. The distribution of two-parameter Weibull distribution is apparently skewed to the right, which is consistent with the histogram density plot superimposed of Von Mises stress data. While, Fig. 6 shows the cumulative function plots for the Weibull distribution and compared with the empirical cumulative distributions of the stress data. From Fig. 6, it can be seen that the two-parameter Weibull distribution is closer to empirical distribution that was obtained on the basis of the measured Von Mises stress data at the node.

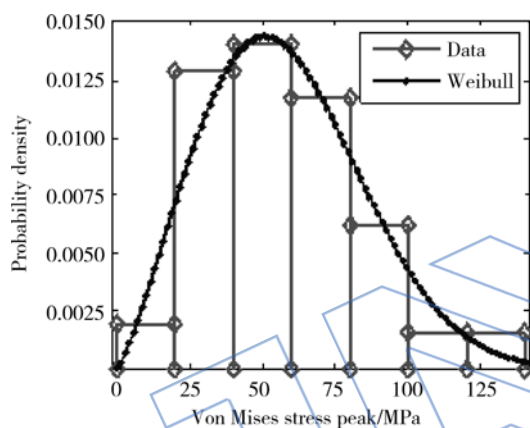


Fig. 5 Probability density function of the Von Mises stress at node 2632

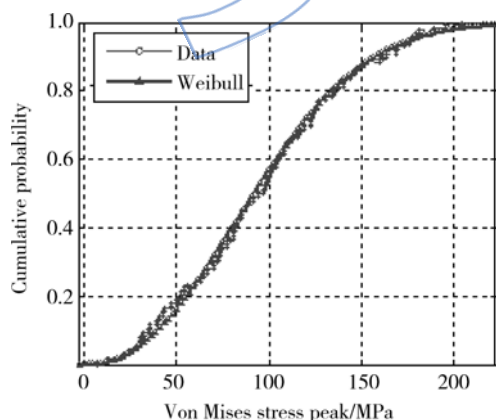


Fig. 6 Cumulative distribution plot of the Von Mises stress at node 2632

The statistical term D_n is obtained by solving Eqs. (21) and (22), which is 0.045 06. From the table of the K-S test, the value of $D_{n,\alpha}$ is 0.134 03 with the sample size is one hundred,

and the value $\alpha = 0.05$. The tests support the conclusion that the two-parameter Weibull distribution provides a good model for distribution of Von Mises stress data at the node.

Fig. 7 shows a comparison of probability densities between the two-parameter Weibull distribution and Dirlik distribution of the Von Mises stress at node 2632. The distributions of both the two-parameter Weibull distribution and Dirlik distribution are apparently skewed to the right, which are consistent with the Von Mises stress data even though the peak stress of Weibull distribution shows a bit higher than that Dirlik distribution.

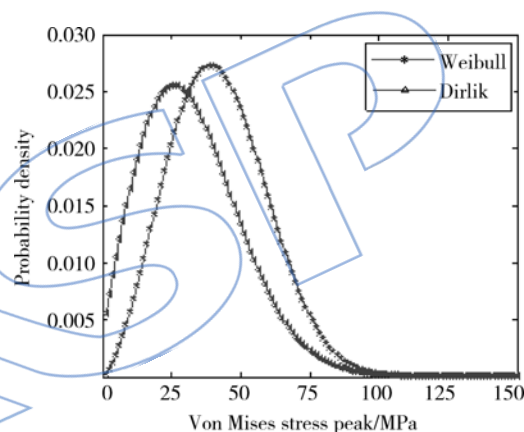


Fig. 7 Comparison of different probability density functions of the Von Mises stress at node 2632

5.4 Fatigue life estimation and result analysis

According to the method for structure sonic fatigue life estimation presented in this paper, using Basquin fatigue life model, computer programs are made to estimate the fatigue life of the combustor liner. Computed results are shown in Table 2. By analyzing the fatigue life estimation results at the typical nodes in the liner structure, the following conclusions can be found: (1) The fatigue life at the node 4793 is shortest under sound pressure level 140 dB, with sound pressure level increases from 140 dB to 160 dB the fatigue life at the node 2632 and 652 is relatively short. Stresses concentrate at flame tube lips, areas near thickened edges and the edges of holes, where fatigue life is relatively short, so special attentions are needed for these key areas

when considering the anti-sonic fatigue design.
(2) Generally speaking, fatigue life of the outer skin is longer than that of the inner skin of the liner. Comparing the results calculated by the Dirlik distribution and the Weibull distribution,

it can be found that the estimated result by Weibull distribution is more conservative than that by the Dirlik distribution, therefore, the estimated fatigue life is more reliable.

Table 2 Calculated fatigue life of the combustor liner by stress probability density methods (unit: cycles)

Node number	Node position	Using Dirlik distribution		Using Weibull distribution	
		140 dB	160 dB	140 dB	160 dB
4947	Primary air hole edge on outer skin	1 252 000	112 600	653 200	54 369.9
499	Dilution air hole edge on outer skin	2 105 000	249 200	111 700	118 370
9372	Rear end of outer skin	2 005 000	200 200	977 800	91 315
652	Primary air hole edge on inner skin	412 400	18 670	226 700	7 325.4
2632	Dilution air hole edge on inner skin	267 800	7 654	140 500	78 475
4793	Rear end of inner skin	223 900	19 800	125 000	10 570
8456	Outer skin of head	258 700	21 680	146 700	10 025

6 Conclusions

1) The probability distribution of Von Mises stress of thin-walled structure under random loadings is suggested to accord approximately with the two-parameter Weibull distribution, and the formula for calculating Weibull parameters are given. K-S test show that the two-parameter Weibull distribution provides a better model for probability distribution of Von Mises stress response.

2) Based on the Miner linear theory, the method for predicting the random sonic fatigue life based on the stress probability density is developed, and the model for fatigue life prediction is constructed.

3) The PSD of the vibrational stress response of an aero-engine combustor liner structure to band-limited Gaussian white noise loading is calculated by using the coupled FEM/BEM model, the fatigue life is effectively estimated in frequency domain by using the constructed model.

4) Comparative analysis shows that the estimated results of sonic fatigue life of the combustor liner structure by using the two-param-

eter Weibull distribution of Von Mises stress are more conservative than that by Dirlik distribution.

References:

- [1] Breitung K, Rackwitz R. Nonlinear combination of load processes[J]. Journal of Structural Mechanics, 1982, 10 (2):145-166.
- [2] Pearce H T, Wen Y K. On linearization points for nonlinear combination of stochastic load processes[J]. Structural Safety, 1985, 2(3):169-176.
- [3] Grigoriu M. Crossings of non-Gaussian translation processes[J]. Journal of Engineering Mechanics, 1984, 110 (4): 610-620.
- [4] Segalman D, Reese G. Estimating the probability distribution of Von Mises stress for structures undergoing random excitation Part 1: derivation [R]. Anaheim, CA, United States: International Mechanical Engineering Congress and Exposition, 1998.
- [5] Segalman D, Reese G, Field R, Jr., et al. Estimating the probability distribution of Von Mises stress for structures undergoing random excitation Part 2: example calculations [R]. Anaheim, CA, United States: International Mechanical Engineering Congress and Exposition, 1998.
- [6] Segalman D, Reese G, Field R, Jr., et al. Estimating the probability distribution of Von Mises stress for structures undergoing random excitation[J]. Journal of Vibration and Acoustics, 2000, 122(1):42-48.
- [7] Petrucci G, Zuccarello B. Fatigue life prediction under wide band random loading[J]. Fatigue & Fracture Engineering

- Materials & Structures, 2004, 27(12): 1183-1195.
- [8] JIN Yishan, LI Lin. Probability density function of Von Mises stress for structures undergoing random excitations [J]. Chinese Journal of Applied Mechanics, 2004, 21(3): 13-16. (in Chinese)
- [9] SHA Yundong, GUO Xiaopeng, ZHANG Jun. Research on the random sonic fatigue life prediction based on the stress probability density and power spectrum density method [J]. Journal of Vibration and Shock, 2010, 29(1): 162-165. (in Chinese)
- [10] Bishop N W M. The use of frequency domain parameters to predict structural fatigue[D]. Coventry, United Kingdom: University of Warwick, 1988.
- [11] Bishop N W M, Sherratt F. Fatigue life prediction from power spectral density data I: traditional approaches[J]. Environmental Engineering, 1989, 2(3): 11-14, 29.
- [12] Bishop N W M, Sherratt F. Fatigue life prediction from power spectral density data II: recent developments[J]. Environmental Engineering, 1989, 2(6): 11-15, 19.
- [13] Zaharim A, Razali A M, Abidin R Z, et al. Fitting of statistical distributions to wind speed data in Malaysia[J]. European Journal of Scientific Research, 2009, 26(1): 6-12.
- [14] Wirsching P H, Light M C. Fatigue under wide band random stresses[J]. Journal of the Structure Division, 1980, 106(7): 1593-1606.

JALSP